Then the local value of **K** is given by

$$\mathbf{K} = (\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \frac{1}{\mu_0}(-3.55\mathbf{a}_x + 3.54\mathbf{a}_y + 4.67\mathbf{a}_z) \times \mathbf{a}_z = \frac{5.0}{\mu_0}\left(\frac{\mathbf{a}_x + \mathbf{a}_y}{\sqrt{2}}\right) \quad \text{(A/m)}$$

13.8. Given $\mathbf{E} = E_m \sin(\omega t - \beta z)\mathbf{a}_y$ in free space, find **D**, **B** and **H**. Sketch **E** and **H** at $t = 0$.

Mathcad

$$\mathbf{D} = \epsilon_0 \mathbf{E} = \epsilon_0 E_m \sin(\omega t - \beta z)\mathbf{a}_y$$

The Maxwell equation $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ gives

$$\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & E_m \sin(\omega t - \beta z) & 0 \end{vmatrix} = -\frac{\partial \mathbf{B}}{\partial t}$$

or

$$-\frac{\partial \mathbf{B}}{\partial t} = \beta E_m \cos(\omega t - \beta z)\mathbf{a}_x$$

Integrating,

$$\mathbf{B} = -\frac{\beta E_m}{\omega} \sin(\omega t - \beta z)\mathbf{a}_x$$

where the "constant" of integration, which is a static field, has been neglected. Then,

$$\mathbf{H} = -\frac{\beta E_m}{\omega \mu_0} \sin(\omega t - \beta z)\mathbf{a}_x$$

Note that **E** and **H** are mutually perpendicular. At $t = 0$, $\sin(\omega t - \beta z) = -\sin \beta z$. Figure 13-7 shows the two fields along the z axis, on the assumption that E_m and β are positive.

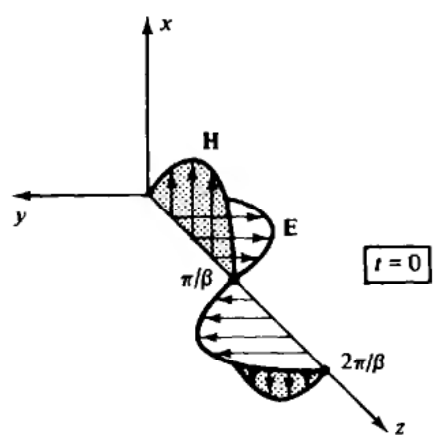

Fig. 13-7

13.9. Show that the **E** and **H** fields of Problem 13.8 constitute a wave traveling in the z direction. Verify that the wave speed and E/H depend only on the properties of free space.

Mathcad

E and **H** together vary as $\sin(\omega t - \beta z)$. A given state of **E** and **H** is then characterized by

$$\omega t - \beta z = \text{const.} = \omega t_0 \qquad \text{or} \qquad z = \frac{\omega}{\beta}(t - t_0)$$

But this is the equation of a plane moving with speed

$$c = \frac{\omega}{\beta}$$

in the direction of its normal, $\mathbf{a}_z$. (It is assumed that β, as well as ω, is positive; for β negative, the direction of motion would be $-\mathbf{a}_z$.) Thus, the entire pattern of Fig. 13-7 moves down the z axis with speed c.

The Maxwell equation $\nabla \times \mathbf{H} = \partial \mathbf{D}/\partial t$ gives

$$\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{-\beta E_m}{\omega \mu_0} \sin(\omega t - \beta z) & 0 & 0 \end{vmatrix} = \frac{\partial}{\partial t} \epsilon_0 E_m \sin(\omega t - \beta z)\mathbf{a}_y$$

$$\frac{\beta^2 E_m}{\omega \mu_0} \cos(\omega t - \beta z)\mathbf{a}_y = \epsilon_0 E_m \omega \cos(\omega t - \beta z)\mathbf{a}_y$$

$$\frac{1}{\epsilon_0 \mu_0} = \frac{\omega^2}{\beta^2}$$

Consequently,

$$c = \sqrt{\frac{1}{\epsilon_0 \mu_0}} \approx \sqrt{\frac{1}{(10^{-9}/36\pi)(4\pi \times 10^{-7})}} = 3 \times 10^8 \text{ (m/s)}$$

Moreover,

$$\frac{E}{H} = \frac{\omega \mu_0}{\beta} = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 120\pi \text{ (V/A)} = 120\pi\ \Omega$$

13.10. Given $\mathbf{H} = H_m e^{j(\omega t + \beta z)}\mathbf{a}_x$ in free space, find $\mathbf{E}$.

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\frac{\partial}{\partial z} H_m e^{j(\omega t + \beta z)}\mathbf{a}_y = \frac{\partial \mathbf{D}}{\partial t}$$

$$j\beta H_m e^{j(\omega t + \beta z)}\mathbf{a}_y = \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathbf{D} = \frac{\beta H_m}{\omega} e^{j(\omega t + \beta z)}\mathbf{a}_y$$

and $\mathbf{E} = \mathbf{D}/\epsilon_o$.

13.11. Given

$$\mathbf{E} = 30\pi e^{j(10^8 t + \beta z)}\mathbf{a}_x \text{ (V/m)} \qquad \mathbf{H} = H_m e^{j(10^8 t + \beta z)}\mathbf{a}_y \text{ (A/m)}$$

in free space, find H_m and B $(\beta > 0)$.

This is a plane wave, essentially the same as that in Problems 13.8 and 13.9 (except that, there, $\mathbf{E}$ was in the y direction and $\mathbf{H}$ in the x direction). The results of Problem 13.9 hold for any such wave in free space:

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ (m/s)} \qquad \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \Omega$$

Thus, for the given wave,

$$\beta = \frac{10^8}{3 \times 10^8} = \frac{1}{3} \text{ (rad/m)} \qquad H_m = \pm \frac{30\pi}{120\pi} = \pm \frac{1}{4} \text{ (A/m)}$$

To fix the sign of H_m, apply $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$:

$$j\beta 30\pi e^{j(10^8 t + \beta z)}\mathbf{a}_y = -j10^8 \mu_0 H_m e^{j(10^8 t + \beta z)}\mathbf{a}_y$$

which shows that H_m must be negative.